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RELATION ORIENTED MODELING FOR HETEROGENEOUS OBJECT DESIGN

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ABSTRACT

Relation oriented modeling approaches are proposed to design heterogeneous objects. The heterogeneous object modeling process is viewed as representing and manipulating complex geometrical, topological and material variation relations with proper data structures. Linear list structure, hierarchical tree structures and more general graph structures are used to represent complex heterogeneous objects. The powerful non-manifold cellular representation and the hierarchical heterogeneous feature tree representation are combined to model complex objects with simultaneous geometry intricacies and compound material variations. We demonstrate that relations play critical roles in heterogeneous object design and under the relation oriented framework, heterogeneous objects can be modeled with generic, uniform representations. The proposed relation oriented modeling approaches are tested with a prototype heterogeneous CAD modeler and presented with different types of heterogeneous object examples.

KEYWORDS: CAD, Heterogeneous object modeling, Functionally graded material, topological relations, non-manifold representation.

INTRODUCTION

Applications of heterogeneous objects have caught considerable focus in CAD, CAE and CAM communities in recent years. In contrast to traditional engineering parts, heterogeneous objects can be made of more than one materials (multi-material objects), with embedded sensors/actuators, with local material compositions and spatially anisotropic properties. With heterogeneous components, multiple and anisotropic properties can be obtained in a single object; abrupt transitions in material compositions or properties can be avoided with gradual material composition or structure variations; therefore material incompatibilities and stress concentrations can be eliminated [1]. It is reported that heterogeneous components have been successfully applied in mechanical, thermal [2], optical [3] and bio-medical [4] fields in recent years.

The wide applications of heterogeneous objects call for systematic approach in heterogeneous object design and a lot of works have been done on this topic in the past decade. These works range from heterogeneous object representations [5] [6] [7] [8], material function design and optimization [9] and object construction tools development [7] [8, 10-12]. In all these studies, heterogeneous object representations play fundamental roles and are crucial to material function design, visualization and other downstream applications. Current heterogeneous object representations are still far from maturation: only simple form heterogeneous objects can be represented and most of the existing approaches lack the modeling capacity for complex heterogeneous objects.

This paper generalizes some of our earlier works on heterogeneous object representations (including the source based representation [7], the hierarchical heterogeneous feature tree based representation [8] and the non-manifold cellular representation [13] and consider the heterogeneous object modeling under a consistent relation-oriented framework. With such a relation oriented vision in heterogeneous object modeling, complex heterogeneous objects can be generally classified into three types according to the underlying data structures in these representations: (I) Objects with intricate geometries and simple material distributions; (II) Objects with compound (2D or 3D dependent) material distributions, but simple or regular geometries: (III) Objects with intricate geometries as well as complex material distributions. In this paper, all these objects are represented with the relation-oriented approaches; detailed data structures for each type of relation based modeling are discussed as follows.

RELATION ORIENTED MODELING FOR HETEROGENEOUS OBJECT DESIGN

In traditional solid modeling, the notion of relation refers to the topological relations $R(G)$ between geometrical entities, e.g. the connectivity and adjacency relations in boundary representation. In the context of heterogeneous object modeling, the relations under consideration include both the topological relations between geometric features as well as the material variation dependencies between heterogeneous features (a heterogeneous feature is an entity with heterogeneous material distributions [8]), as formulated by Eq.(1).

$$R = \{R(G), R(M), R(G, M)\} \quad (1)$$

where $R(G)$ denotes the geometrical/topological relations, $R(M)$ refers to the relations used for representing the material distributions, and $R(G, M)$ denotes the relations coupled between $R(G)$ and $R(M)$.

Since there have been mature data structures for describing topological relations, e.g. the well known manifold boundary representation [14] and the non-manifold radial edge data structures [15], most of existing efforts try to extend traditional solid modeling approaches with emphasis on material distributions, and $R(M)$ is regarded as the focus. $R(G, M)$, the relation coupled between $R(G)$ and $R(M)$ has been rarely addressed before. Such an isolation of the relation between the geometry information and material variation information naturally leads to a sequential approach [8] in heterogeneous object modeling. As a consequence, the modeling capacities, especially the representational ability for complex heterogeneous objects are usually limited. We will explain how certain simplicity occurs in these objects modeled with "linear relations" and the "hierarchical relations", and how $R(G, M)$ based modeling can help to design more complex heterogeneous objects in the following Sections.

MODELING OBJECTS WITH SIMPLE MATERIAL VARIATIONS BASED ON LINEAR RELATIONS

Siu and Tan [7] proposed a source based method for modeling heterogeneous objects with functionally graded material (FGM) distributions. In their method, the geometry features (termed as "sources") from which the material composition varies are saved in a list structure, as shown in Fig.1. The geometric representation used in this method is traditional boundary representation, and the material distribution is represented by the sources features, in essence, by the relations $R(M)$ between the sources.

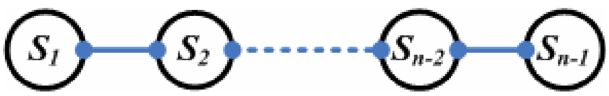


Fig.1. Modeling linear relation with a list structure

By evaluating the distances from all the source features, the material composition at a specific location can be evaluated at runtime:

$$M(P) = \sum_{i=0}^{n-1} W_i M_i \quad (2)$$

where $M(P)$ is the material composition for an arbitrary point P inside the object, n is the total numbers of the source entities in the list structure, M_i is a homogeneous material associated with the source S_i, W_i is the material blending weight for source S_i .

Note that some methods implicitly keep the "sources" and assume that the material variation source features are already known or constructed in advance. For example, Biswas etc. [9] model heterogeneous objects with distance fields, and their concept of "material features" share the same concept as Siu's source features [7]. In both methods, the underlying data structure can be generally represented by the linear relation shown in Fig.1.

Fig.2. shows an example heterogeneous object modeled with source based scheme. The left (S1) and right plane (S2) are saved as the source features in the list, and a linear material gradation from S1 to S2 is modeled.

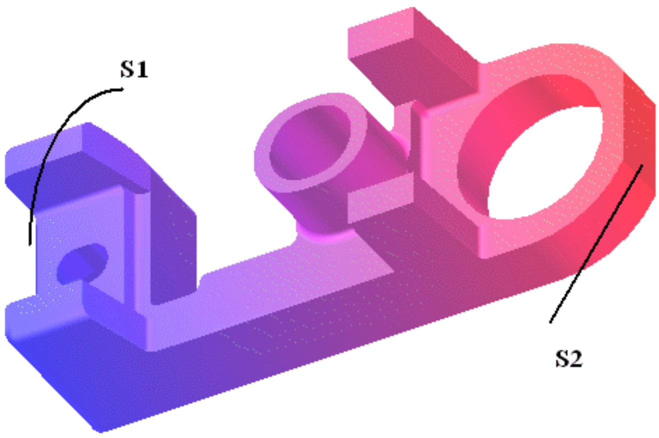


Fig.2. An example heterogeneous object with a linear gradation

As is seen from Fig.2, the example heterogeneous object is complex in geometries, however the material distributions defined over the complex geometry remains simple.

Although objects with simple material gradations are easy to model and fabricate, however, their functional performances are often limited. In some applications, compound material variations (two or more simultaneous material gradations) are preferred, for example, the temperature distributions of modern aerospace shuttles have variations in two or three directions [16]; if the FGM has two-dimensional dependent material properties, more effective high-temperature resistant material can be obtained. However, as the source based scheme is based on linear data structures (e.g. array, list or other equivalent structures), therefore possible relations that can be modeled are usually limited to simple forms. Consequently these methods fail to represent compound material variations. Based on this observation, to model more complex material distributions, necessary data structures for material modeling need to be extended.

A HIERARCHICAL RELATION BASED MODELING FOR COMPOUND MATERIAL VARIATIONS

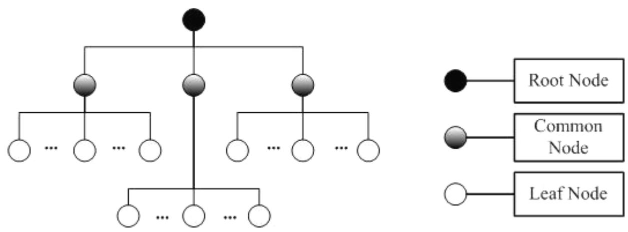


Fig.3. Modeling hierarchical relations with a tree structure

The hierarchical relation is more complex than linear relations in describing $R(M)$ and has been successfully applied to modeling compound material variations. Linear list or array structures are replaced with tree structures, as is shown in Fig.3.

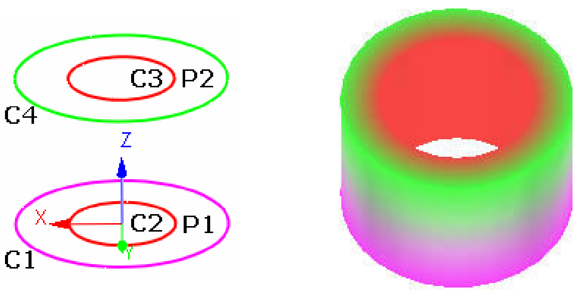
A Heterogeneous Feature Tree (HFT) structure is proposed to model compound material variations [8]. Each HFT is composed of a collection of nodes, in which each node may also have a collection of child nodes. The HFT is defined to maintain the material variation dependencies with different hierarchies: the material composition of a feature in a higher level is dependent on the material composition of its child features; and the material composition evaluated from each child trees are then blended at their parent level in the material evaluation process, as shown in Eq.(3):

$$M(P) = M^{(0)}(P) = \sum_{i=0}^{N_0-1} W_{(i)}^{(1)} M_{(i)}^{(1)}(P)$$
$$M_{(i)}^{(j)}(P) = \sum_{i=0}^{N_j-1} W_{(i)}^{(j+1)} M_{(i)}^{(j+1)}(P) \qquad (3)$$

where $M^{(0)}{}_{(P)}$ is the material composition evaluated at the root node of the HFT; $M_{(i)}{}^{(j)}{}_{(P)}$ is the material composition evaluated from the i-th child node of a given node at hierarchy level j; N_j is the total number of children counts of a given node at level j;

Eq.(3) shows that the material evaluation based on the HFT representation is a recursive process. If a node in a HFT is a leaf node, then the material distribution it represents is defined to be homogeneous; otherwise, the material composition are evaluated from lower hierarchies to higher ones.

Samanta and Koc [11] also proposed using B-Spline surfaces and B-Spline volumes to represent 2D and 3D-dependent material variations, which, from the relation oriented perspective, is also based on the hierarchal representation, although such relations are implicitly kept. Other similar works can be also found in some recent papers, for example, Martin and Cohen [17], J. Hua et. al. [18] suggested using trivariate splines to represent and specify volumetric attributes. Natekar et. al. [19] utilized NURBS volume to represent the geometry as well as the field definitions in constructive solid analysis.



(a) (b)

Fig.4. An example heterogeneous object with a compound material distribution. (a) Constructive heterogeneous features; (b) Output heterogeneous object

Fig.4 illustrates an example heterogeneous object with compound material variations. Corresponding heterogeneous feature tree structure used for representing material variation dependencies is shown in Fig.5.

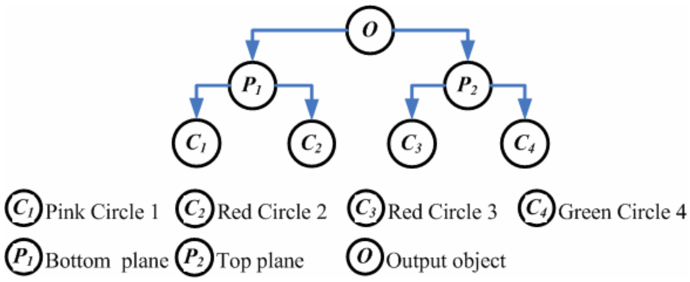


Fig.5 Hierarchical relation modeled with heterogeneous feature tree structure

From the comparison of Fig.1 Fig.2, Fig.4 and Fig.5, it can be found that:

(1) The source features in source based approach are assumed to be homogeneous; while in the HFT based representation, each source may be heterogeneous as well;
(2) Each heterogeneous source feature may contain their own “source” feature lists (with their own lists or arrays); so the relation modeled with source based approach can be regarded as a special case in the HFT representation, in which only one hierarchy exists.
(3) The geometries represented with the hierarchical representation are of simple forms. Although the hierarchical representation can be also used to model compound material variations for objects as Fig.2, however, due to the geometry intricacies of such objects, it is non-trivial to avoid conflicting/overlapping or unspecified material definitions for different parts of the complex object.

Usually the hierarchical relation based methods are good at modeling heterogeneous objects with complex material variations, however, the geometries of these objects are usually of simple forms, for example extruded, swept or lofted objects only [8] [11]. From the relation oriented perspective, this is because the hierarchical tree structure is designed to represent the material variation only and the geometrical/ topological relations of the objects are not considered at all.

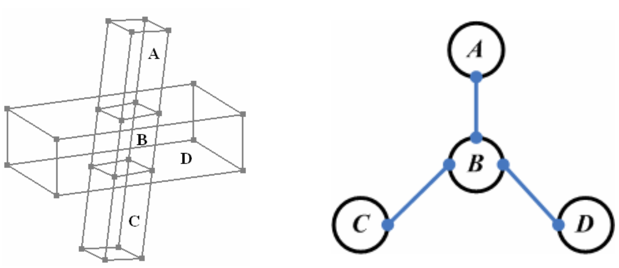
By incorporating both the topology information and the material variation relations in the same model, we introduce a class of more general relations for modeling complex objects, which allow simultaneous geometry intricacies and compound material variations.

GRAPH BASED RELATION MODELING FOR TRULY COMPLEX HETEROGENEOUS OBJECTS

To model heterogeneous objects with complex geometries and compound material variations, the hierarchical heterogeneous feature tree structure is extended and a general graph structure is proposed [13]. Fig. 6 (a) illustrates an example complex object which is represented by non-manifold quasi-disjoint cellular representation [13].

$$
\begin{aligned}
&O = (G, M) = \{A, B, C, D\} \\
&A = (G^{(A)}, T^{(A)}), B = (G^{(B)}, T^{(B)}), \\
&C = (G^{(C)}, T^{(C)}), D = (G^{(D)}, T^{(D)}) \\
&G = G^{(A)} \cup G^{(B)} \cup G^{(C)} \cup G^{(D)} \\
&Dim(G) = Dim(G^{(A)}) = Dim(G^{(B)}) = Dim(G^{(C)}) = Dim(G^{(D)})
\end{aligned} \tag{4}
$$

where $\cup$ denote the Boolean union, *Dim*(.) returns the object's dimension. *A*, *B*, *C* and *D* are called heterogeneous cells, G and T refers to the geometry and heterogeneous feature tree structures of the object or the cell. The cells {*A*, *B*}, {*B*, *C*}, {*B*, *D*} are adjacent, while cells {*A*, *C*} are mutually disjoint. Fig. 6 (b) shows the cell adjacency status for this object, two cells are linked with an arc if they are adjacent.



(a) (b)

Fig.6 An example complex object; (a) Non-manifold cellular representation; (b) Cell adjacency

Note that in this representation, each cell has its own HFT structures in their material distributions, rather than a single HFT for all cells. This makes it possible to represent complex material variations through decompositions. Each decomposed subset (i.e. a heterogeneous cell) can have any compound material distributions as depicted in previous sections. Also note that the hierarchical relations used for each cell are not independently defined, as is graphically illustrated in Fig.7.

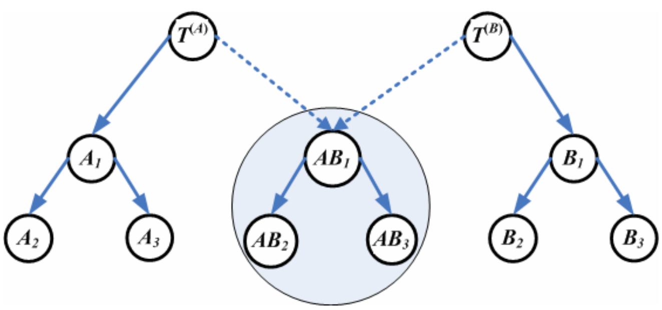


Fig.7. Complex relation modeled with general graph structure

Two adjacent heterogeneous cells may share some common elements (HFT node or HFT branches) in their HFT structures. As seen in the hierarchical representation, the HFT represents the material distributions, so sharing the HFT nodes/branches guarantees that the adjacent cells share some common material definitions or distributions. In other words, the smooth material transitions between the adjacent cells are assured, and in turn the overall smooth material transitions throughout the complex heterogeneous objects can be guaranteed [13].

In our original heterogeneous feature tree representation [8], each node may have many child nodes, but only one parent node. In Fig.7, the node AB_1 points to two parental nodes, as indicated by the dot line arrows. This extends previous hierarchical relations into a class of more general relations in which sharing information is incorporated in addition to the hierarchical dependency relations. In this paper, such complex relations coupled between *R*(*G*) and *R*(*M*) are generally represented by graph structures, as shown in Fig.7. In contrast with the proposed linear relation [7] and the hierarchical relation [8], in this graph based representation [13], the topological adjacency information between the cells is also utilized in the material modeling process; the complexity of geometry as well as that of material distributions can be considered within the same model. Geometric modeling and material modeling are seamlessly integrated rather than separately processed. This avoids traditional sequential modeling (i.e. geometric modeling followed by material modeling) and is intuitive to capture the designers' intentions.

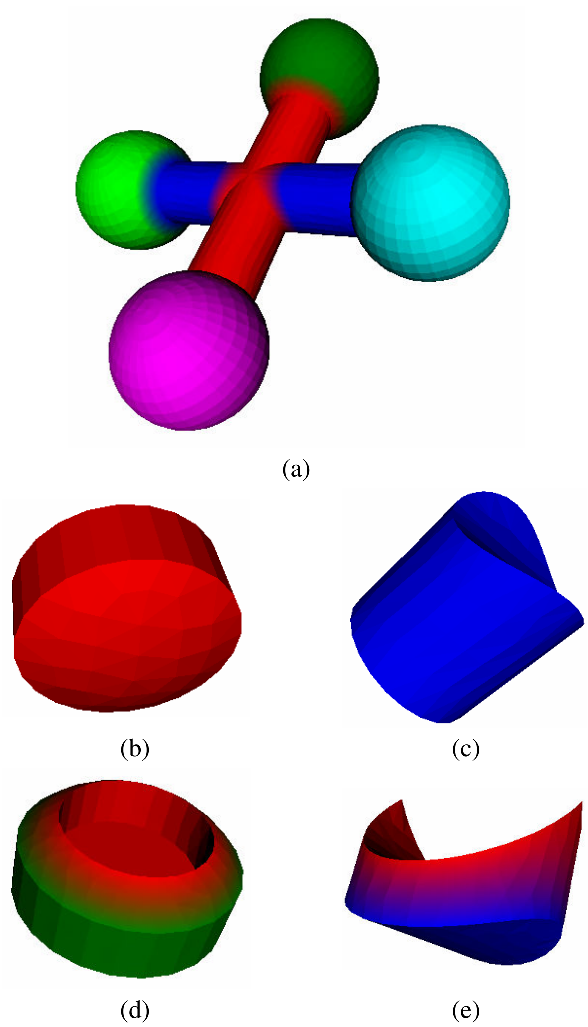


Fig.8 Complex heterogeneous object modeled with a graph structure. (a) Full object; (b)-(e) Four compositional heterogeneous cells

Fig. 8 illustrates a 3D example modeled with heterogeneous Boolean operations [13]. A heterogeneous Boolean operator is defined to include two parts of operational directives: geometrical directive and material variation directive. The geometrical directive can be any traditional Boolean operators, e.g. union, intersection or subtraction; and the material variation directive is proposed to control different material variation layouts during the heterogeneous Boolean operation [13]. Two homogeneous cylinders are first unioned and the then four spheres are sequentially joined with object. The material variation directive is used to impose smooth material variations throughout the complex object. Notice that two homogeneous objects unioned with Boolean operations can yield functionally graded layers around the intersection regions, as is amplified in Fig.8.(d) and Fig.8.(e). This shows inter-influences from the primitive objects which reflects both geometrical (geometry union) and material relations (material gradient) that underlies the CAD model.

IMPLEMENTATIONS AND EXAMPLES

All the proposed relation oriented approaches have been implemented in our prototype software package *CAD4D*. *CAD4D* [20] is an interactive heterogeneous object modeler developed by the authors at the department of Mechanical Engineering, The University of Hong Kong. Fig. 9 shows the graphic user interface of *CAD4D*.

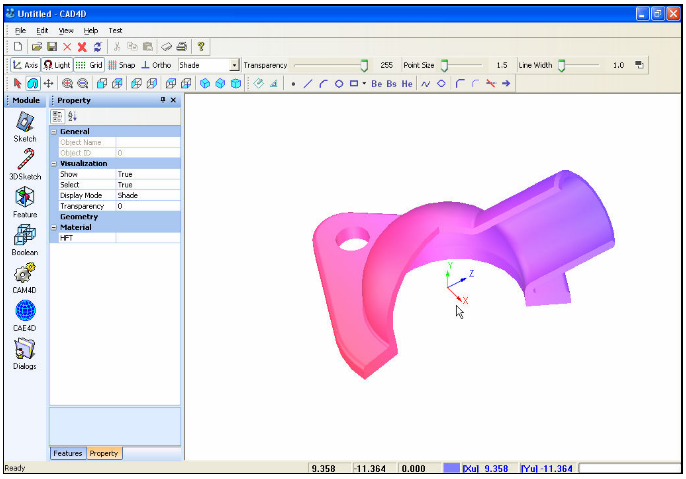


Fig. 9 GUI of *CAD4D*, an interactive heterogeneous CAD modeler

We have also developed collections of 1D, 2D and 3D heterogeneous features based on the proposed data representations, which are encapsulated in a heterogeneous object library—*Lib4D*, with an open architecture for extension and development. *Lib4D* is an object class library containing over 10,000 code lines, written in C++ language in Windows platform. *Lib4D* provides convenient interfaces for heterogeneous model creation, modification and visualizations. Fig.10 illustrates part of the object hierarchies in *Lib4D*. As examples, two complex objects modeled with the relation oriented approach are also illustrated in Fig.11.

CONCLUSIONS

Several relation oriented representations are reviewed and the vision for relation oriented modeling in heterogeneous object design has been explained with examples and detailed data structures. The relations are defined to include the topological relations between geometric features, the material variation dependencies between heterogeneous features, and also the relation coupled between the topological information and material information. We consider the heterogeneous object modeling process as representing and manipulating complex geometrical, topological and material variation relations. From the linear list structure, hierarchical tree structures to the more general graph structures, it can be concluded that complex relations can help to represent complex heterogeneous objects with simultaneous geometry intricacies and compound material variations.

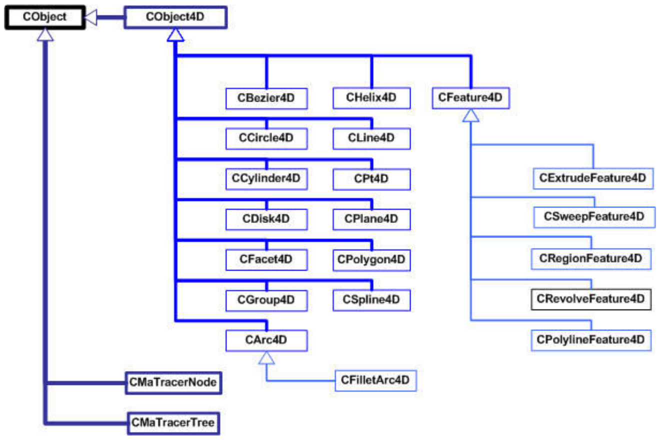
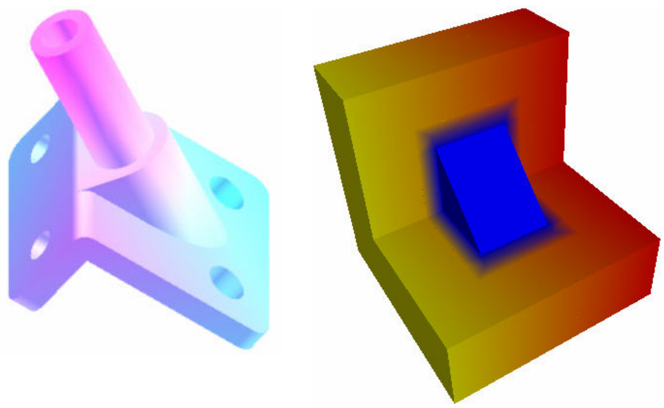


Fig. 10 Part of object hierarchies in *Lib4D*



(a) (b)

Fig.11 Some 3D examples (a) Modeled with hierarchical heterogeneous feature tree representation; (b) An object with smooth material transitions unioned from a linear graded block and a homogeneous block.

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REFERENCES

[1] S. Suresh and A. Mortensen, *Fundamentals of functionally graded materials: processing and*

thermomechanical behaviour of graded metals and metal-ceramic composites. London: IOM Communications Ltd, 1998.

[2] X. Qian and D. Dutta, "Design of heterogeneous turbine blade," *Computer-Aided Design*, vol. 35, pp. 319, 2003.

[3] Uemura, "The Activities of FGM on New Application," presented at Functionally graded materials VII, Proc. 7th Int. Symposium on Functionally Graded Materials, Materials Science Forum, Beijing, China, 2002.

[4] H. K. e. Fumio Watari, "Development of Functionally Graded Implant and Dental Post for Bio-Medical Application," presented at Functionally graded materials VII,Proc. 7th Int. Symposium on Functionally Graded Materials, Materials Science Forum, Beijing, China, 2002.

[5] T. R. Jackson and W. Cho, "Analysis of Solid Model Representations for Heterogeneous Objects," *JCISE: Journal of Computing and Information Science In Engineering, ASME Transactions*, vol. Vol.2, pp. Page1-10, 2002.

[6] V. Adzhiev and E. Kartasheva, "Cellular-functional modeling of heterogeneous objects," *Proceedings of the seventh ACM symposium on Solid modeling and applications, Germany*, 2002.

[7] Y. K. Siu and S. T. Tan, "'Source-based' heterogeneous solid modeling," *Computer-Aided Design*, vol. 34, pp. 41, 2002.

[8] X. Y. Kou and S. T. Tan, "A hierarchical representation for heterogeneous object modeling," *Computer-Aided Design*, vol. 37, pp. 307, 2005.

[9] A. Biswas, V. Shapiro, and I. Tsukanov, "Heterogeneous material modeling with distance fields," *Computer Aided Geometric Design*, vol. 21, pp. 215, 2004.

[10] K.-H. Shin, "Representation and process planning for layered manufacturing of heterogeneous objects," PhD Thesis, University of Michigan, 2002.

[11] K. Samanta and B. Koc, "Feature-based design and material blending for free-form heterogeneous object modeling," *Computer-Aided Design*, vol. 37, pp. 287, 2005.

[12] W. Sun and X. Hu, "Reasoning Boolean operation based modeling for heterogeneous objects," *Computer-Aided Design*, vol. 34, pp. 481, 2002.

[13] X. Y. Kou, S.T.Tan, and W. S. Sze, "Modeling truly complex heterogeneous objects with Boolean operations," *Technical report MECAD0501, Department of Mechanical Engineering, The University of Hong Kong*, 2005.

[14] M. E. Mortenson, *Geometric modeling*, 2nd ed. ed. New York: Wiley, 1997.

[15] K. J. Weiler, "Topological structures for geometric modeling," PhD Thesis, Rensselaer Polytechnic Institute, 1986.

[16] M. Nemat-Alla, "Reduction of thermal stresses by developing two-dimensional functionally graded materials," *International Journal of Solids and Structures*, vol. 40, pp. 7339, 2003.

[17] W. Martin and E. Cohen, "Representation and extraction of volumetric attributes using trivariate splines: a mathematical framework," *Proceedings of the sixth ACM symposium on Solid modeling and applications*, pp. 234-240, 2001.

[18] J. Hua, Y. He, and H. Qin, "Multiresolution Heterogeneous Solid Modeling and Visualization Using Trivariate Simplex Splines," *Proceedings of the Ninth ACM Symposium on Solid Modeling and Applications, Genova, Italy*, pp. 47 - 58, June 2004.

[19] D. Natekar, X. Zhang, and G. Subbarayan, "Constructive solid analysis: a hierarchical, geometry-based meshless analysis procedure for integrated design and analysis," *Computer-Aided Design*, vol. 36, pp. 473, 2004.

[20] X. Y. Kou and S. T. Tan, "An interactive CAD environment for heterogeneous object design," *Proceedings of ASME 2004 Design Engineering Technical Conferences, September 28-October 2, 2004, Salt Lake City, Utah USA*, 2004.