

Heterogeneous Object Design: An Integrated CAX Perspective

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Abstract. *CAD modeling, analysis of properties and fabrication* of heterogeneous objects have been extensively studied in the past few decades. Conventionally these topics are separately investigated in CAD, CAE and CAM communities. Such explicit separations, however, suffer from some apparent limitations. This article presents an alternative scheme to consider the heterogeneous object design problem in an integrated CAX (CAD/CAE/CAM) framework. The motivation is to design heterogeneous objects which not only *look right*, but also *functionally work right*. In addition to the data representation, model constructions and visualizations, other considerations such as data communications to and from CAE/CAM modules, fabrication efficiency in layered manufacturing etc., are also considered. The presented CAX based design method facilitates design tools integration and enhances the interoperability in the entire design process.

1 Introduction

Tremendous research efforts have been made in modeling objects with designed material heterogeneities. Throughout the whole design process, the following questions are frequently raised by the end users: (1) How to represent the heterogeneous objects' geometries and material distributions? (2) Does the designed object fulfill the users' functional requirements? How to validate its functional properties? and (3) Is it physically realizable? How to make it? Conventionally these topics are separately investigated in CAD, CAE and CAM communities; however, such explicit separations suffer from some apparent limitations. CAD modelers are usually unable to ascertain whether the modeled objects can really meet the end users' functional requirements, as they only concentrate on the data representations, model constructions and object visualizations. CAE engineers focus on using analytical and numerical approaches to simulate the behaviors of the objects, however, due to the lack of powerful CAD models, only objects with simple (e.g. unidirectionally graded [1]) material distributions were vigorously studied [2-6]. Separate studies in CAD and CAM of heterogeneous object also impede the interoperability required at the process planning and fabrication stages: the data structures of the CAD models were seldom fully utilized to improve the manufacturing efficiency and product qualities, only direct one to one data conversions are conducted, resulting in degraded fabrication performances or productivities.

Traditional design approaches emphasized the modularity and maintainability of heterogeneous object design; however they failed to answer *all* the questions the end users are concerned with. From a practical point of view, the answers to *all* of these questions are indispensable to assure the design qualities and failures in either one may

undermine the design feasibility and authenticity. This article is motivated to address the heterogeneous object design from an integrated collaborative perspective. In addition to the modular design methodologies, we emphasize the data communications and effective use of CAD models in the downstream CAE and CAM modules.

The subsequent sections of this article are organized as follows. CAD modeling of heterogeneous objects is first described in Section 2, where the key concept and usage of our extended Heterogeneous Feature Tree (eHFT) structure are presented. Based on the eHFT model, the Finite Element Analysis (FEA) and Rapid Prototyping (RP) of heterogeneous objects are discussed in Section 3 and Section 4. Section 5 describes the implementation details of the integrated CAX approach and finally concluding remarks and discussion are offered in Section 6.

2 CAD Modeling of Heterogeneous Object

CAD modeling of heterogeneous objects received considerable research focus in the past and there have been a variety of models in the literature. Among them, the voxel model [7, 8], volume mesh model [9], implicit function model [10, 11], explicit function model [3, 4], control point based model [12, 13], control feature based model [14-18], assembly model [19, 20], cellular model [21, 22] and composite hybrid model [23, 24] are most widely used [25]. In this article, we utilize the Heterogeneous Feature Tree (HFT) structure and a Heterogeneous Cellular Representation (HC-Rep), which fall in the scope of the *control feature based model* and the *cellular model*, to present our integrated CAX perspective on heterogeneous object design.

2.1 The Heterogeneous Feature Tree (HFT) Representation

Heterogeneous objects are generally characterized as having location *dependent* material compositions [17]. The idea of the HFT representation is to represent the material

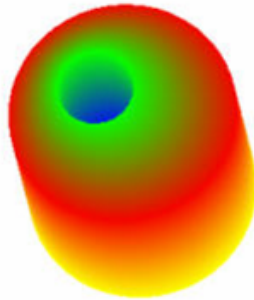


Fig. 1. A heterogeneous object with bi-directional material gradations¹

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heterogeneities by encoding the material variation *dependencies* with a proper data structure. A tree structure is selected because of its hierarchical nature [18] and the capability of representing complex (e.g. 2D or 3D dependent) material heterogeneities.

For instance, the heterogeneous object in Fig. 1, constructed sequentially as shown in Fig. 2, can be conceptually represented by a simplified heterogeneous feature tree structure as shown in Fig. 3.

The Heterogeneous Feature Tree (HFT) structure maintains the material variation dependencies with different hierarchies [18, 26]. By definition, the material composition of a feature in a higher level is dependent on (or determined by) its child features²

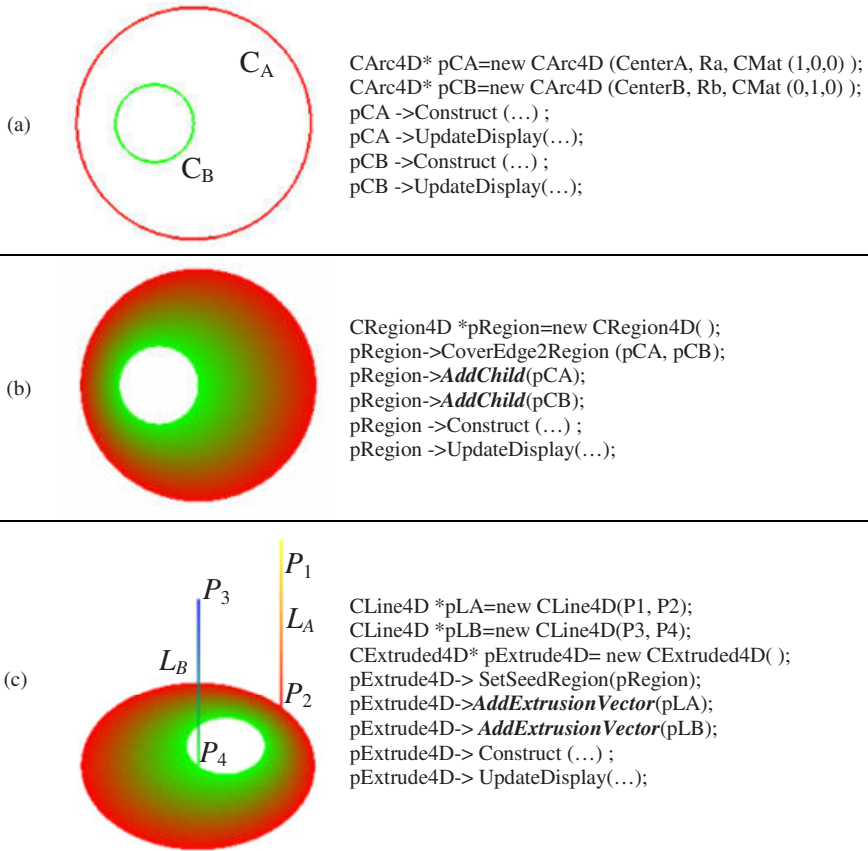


Fig. 2. The modeling process and the pseudo code for the heterogeneous object construction. The bold italic function calls are subroutines used for the construction of the heterogeneous feature tree structures, reprinted from [18] with permission from Elsevier².

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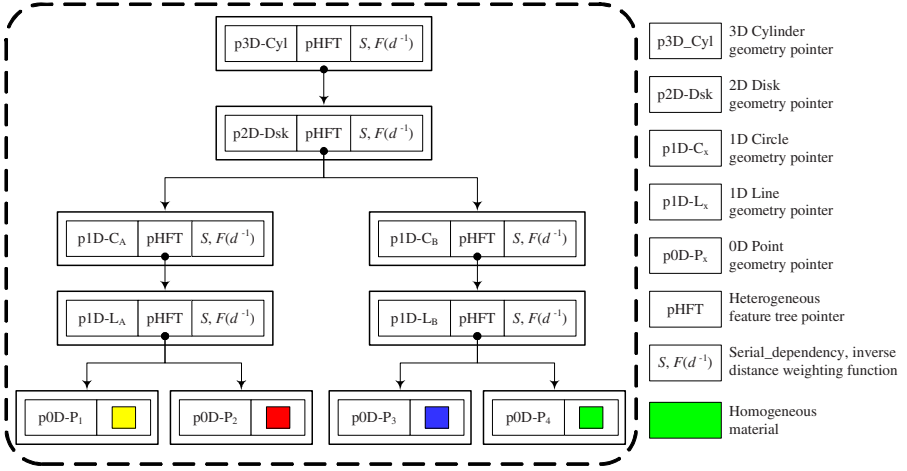


Fig. 3. A simplified HFT representation for the object in Fig. 1. Colors are used to represent different materials

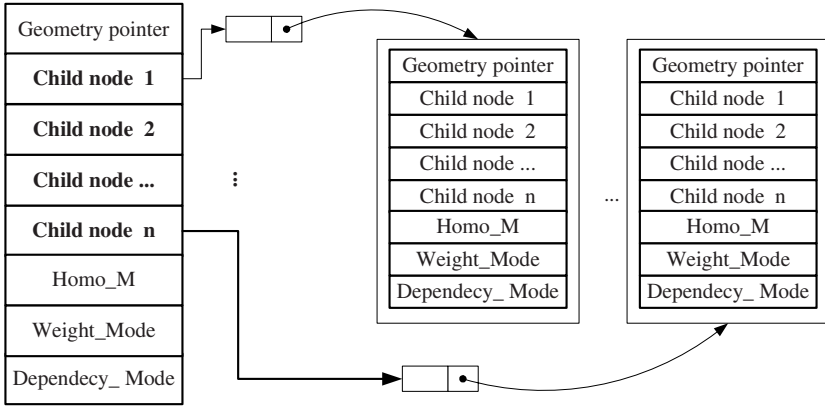


Fig. 4. The Heterogeneous Feature Tree (HFT) structure, reprinted from [18] with permission from Elsevier³

geometries and material definitions. The material compositions evaluated from each child tree are interpolated/blended at their parent level, and are used to represent the parent feature's material distributions [26].

For an arbitrary point P inside this heterogeneous cylinder, its material is dependent on the base 2D region's material composition (since the object is directly extruded from the 2D region, see Fig. 2 (c)). Each section perpendicular to the extrusion vector is a

³ The figure was reprinted from Computer-Aided Design, 37(3), Kou, X.Y. and S.T. Tan, *A hierarchical representation for heterogeneous object modeling*, pp. 307-319, Copyright (2005), with permission from Elsevier.

heterogeneous 2D region, which is similar to the base 2D region in both geometry and material distributions. For each section, the material composition is determined by two bounding contours, (Fig. 2 (a)). Along the extrusion directions, these contours' material variations are constrained (regulated) by the other two heterogeneous vectors (L_A and L_B , Fig. 2 (c)). The material distribution of these extrusion vectors, in turn, are dependent on the composition of their bounding vertices (i.e. P_i , $i=1, 2, 3, 4$).

It can be seen that the constructed HFT structure in Fig. 3 faithfully conveys such material variation dependencies as described above. Note that the HFT structure in Fig. 3 is only a graphic representation which qualitatively depicts the material variation dependencies across different hierarchies. The complete HFT structure also embraces other information such as blending functions between child-parent features and other enumeration data, as shown in Fig. 4.

2.2 Extended Heterogeneous Feature Tree (eHFT) Structure and the Heterogeneous Cellular Representation (HC-Rep)

The HFT representation is initially proposed to model 2D or 3D dependent material gradations with hierarchical tree structures [18]. However, in real application, only few objects have such regular material gradations. It is commonplace that hybrid homogeneous, Functionally Graded Material (FGM) and other distributions coexist in different portions of a complex heterogeneous object. Fig. 5 demonstrates an example of such object. For such objects, the HFT representation is still inadequate.

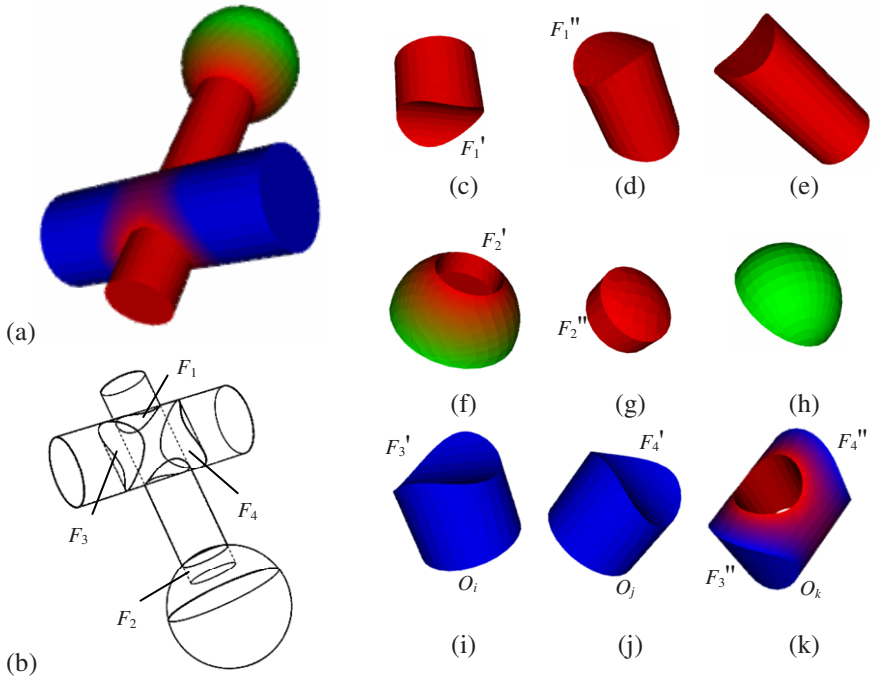


Fig. 5. A heterogeneous object composed of multiple components

To remedy this, a natural idea is to introduce multiple HFT structures to model objects with complex heterogeneities. Using the part-assembly model to represent the object geometries and associating each part with a HFT structure seem to be an intuitive solution. However, the part-assembly model introduces serious data redundancies and inconsistency problems (as will be further elaborated in Section 4). For instance, the faces F_i' and F_i'' ($i=1, 2, 3, 4$) in Fig. 5 represent exactly the same geometries, however are repetitively kept in separate parts. From the visualization point of view, such data redundancies do not matter much, however, when the model undergoes further manipulations (for instance, local translation or deformations of F_i), inconsistent and self-intersected geometries may occur. This is because F_i' and F_i'' are separately translated or deformed and there is no guarantee that they will be exactly identical, especially when the computation error or other noise sources are also involved.

Using multiple, independent HFT structures to represent the material distribution also suffer from similar problems. For instance, if one of the component's material (e.g. Fig. 5 (j)) is changed to another material, its neighbor component's material distributions will not change accordingly (Fig. 5 (k)), resulting in sharp material transitions and possibly, stress concentrations.

Our solution to this problem is to use a novel Heterogeneous Cellular Representation (HC-Rep) to represent complex heterogeneous objects. A heterogeneous object is defined as a collection of heterogeneous cells, each of which, graphically, resembles the parts shown in Fig. 5 (c) to (k). However, the part-assembly model is substituted with the *non-manifold cellular model* and *extended HFT* structures [22] are utilized to model the material heterogeneities.

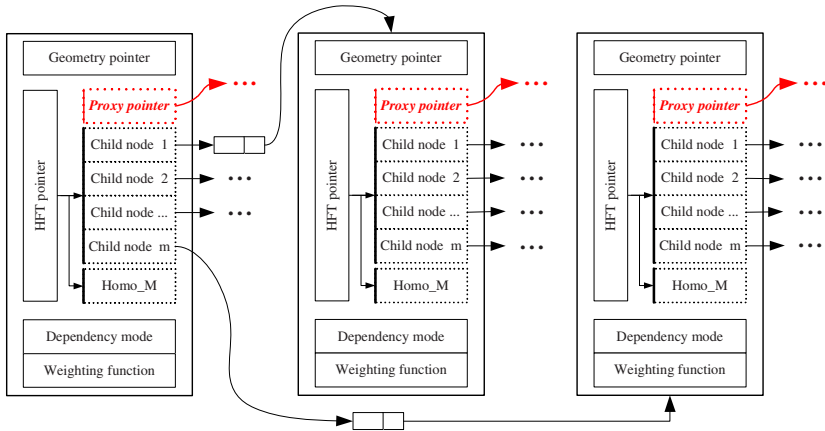


Fig. 6. The extended Heterogeneous Feature Tree structure (eHFT), reprinted from [22] with permission from Elsevier⁴

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The non-manifold cellular model uses both the oriented faces (single-sided) and the ‘double-sided’ boundary faces (co-boundaries) in the computer model. As is shown in Fig. 5 (b), the faces F_i ($i=1, 2, 3, 4$) which delimit the complex object are modeled as *co-faces* and are *shared* between adjacent cells (rather than explicitly copied to each cell). This naturally solves the aforementioned data consistency problem because the applied transformations will be exerted on the shared entities. The computation error, if exists, has the same impact on both cells and there will be no contradictory geometries (e.g. self-intersections) in the modified computer model.

Based on a similar entity-sharing mechanism, the extended HFT (eHFT) structure is proposed to model the local (cell level) as well as the overall (object level) material distributions. The eHFT is characterized as sharing part of the tree branches with other HFT structures. To enable this HFT sharing, the Proxy-HFT (PHFT) is proposed, as shown in Fig. 6. The PHFT points to an existing heterogeneous feature in the modeling space. The feature that is pointed to by the PHFT pointer is called a *proxy*, and the cell that uses the PHFT is called a *client* [22]. Based on this proxy-client mechanism, the material composition evaluation for a cell can be directly *forwarded* or *delegated* to its proxy features. If a client feature contains a valid proxy HFT pointer as shown in Fig. 6, the material composition for a point inside or on the client’s boundary is dynamically determined by calling the proxy feature’s material evaluation procedure. Otherwise if the proxy feature pointer is a NULL pointer, then it degenerates to the common HFT representation as previously discussed; and the material evaluation can be executed according to the encoded hierarchical dependencies.

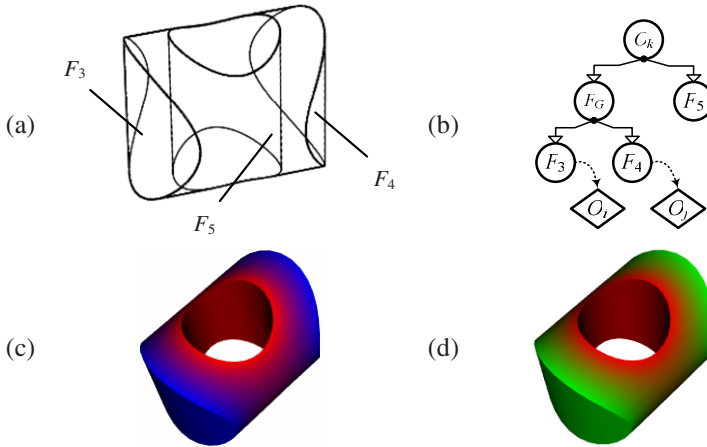


Fig. 7. The eHFT representation for the cell in Fig. 5 (k)

Fig. 7 shows the eHFT representation for the cell in Fig. 5 (k), whose material distribution is defined as a gradation between the face (F_5) and the two co-faces (F_3 and F_4), while the material distributions of F_3 and F_4 are not represented by the usual HFT

structures, instead, two proxies (O_i and O_j , see Fig. 5 and Fig. 7) are proposed to ‘share’ their material distributions with the clients (i.e. F_3 and F_4).

The benefits of sharing the HFT branches are obvious: data redundancy is eliminated; modifications on the material distributions are efficiently updated; and smooth material gradations can be guaranteed. For instance, when the proxy O_i ’s or O_j ’s (Fig. 5 (i), (j)) material compositions are changed from “blue” to “green” (see Fig. 7 (c) and (d)), its neighbor O_k ’s material (Fig. 5 (k)) can be immediately reflected due to the feature sharing mechanism. Note that this auto-update ability allows for local material editions to be properly propagated to the entire heterogeneous object, and this is crucial to avoid abrupt material transitions in FGM object modeling.

3 Finite Element Analysis (FEA) of Heterogeneous Object

The previous section focuses on CAD modeling of heterogeneous objects. Based on the CAD models, other specific modeling tools can be developed to facilitate users to design objects with the *desired* geometries and material distributions. Nevertheless, the word “desired” here mostly refers to visual appearances of the objects. A visually pleasing heterogeneous object may *look right*, however, there is no guarantee that it can *functionally work right*. To assure the designed object can properly work as required, finite element analysis and other numerical approaches can be conducted to evaluate its physical properties or performances.

FEA of heterogeneous objects is a well investigated topic in CAE communities. However, most investigations focus on objects with simple material heterogeneities. For instance, FEA on unidirectionally graded objects account for the majority of case studies in existing literature [2-6]. The primary reason for this phenomenon is not because the finite element method is incapable of handling more complex objects, but possibly due to insufficient support in formulation/representation of complex material heterogeneities. Also note that contemporary CAD modelers (e.g. Solidworks,

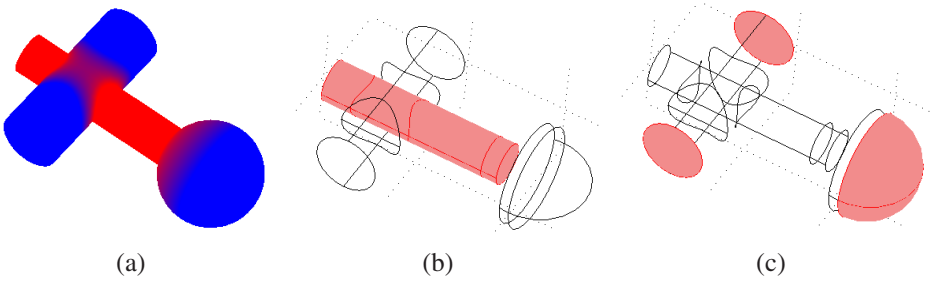


Fig. 8. A steady state thermal conduction analysis. (a) The heterogeneous material distributions of the object under examination; (b) Boundary condition (I), highlighted surfaces are constrained at a temperature of 773.15K; (c) Boundary condition (II), highlighted surfaces are constrained at a temperature of 313.15K.

4. The material properties at sampled points are saved in grid data files, following a prescribed format recognizable to COMSOL;
5. COMSOL imports the grid data files and determine the material properties at FE nodes through interpolations (in this example linear interpolation is used);
6. Boundary conditions are defined within COMSOL;
7. COMSOL computes the local stiffness matrix, assembles global stiffness matrix and solves the nodal variables (temperatures);
8. COMSOL performs post processing to generate graphical outputs.

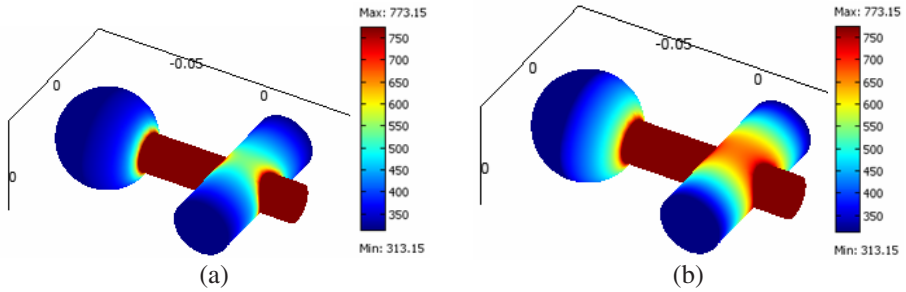


Fig. 10. Results of the steady state thermal conduction analysis. (a) Temperature distribution of an FGM object (gradation between Aluminum $k=155$ W/m/K and Alumina $k=27$ W/m/K); (b) Temperature distribution of a homogeneous object (Aluminum), unit in (K).

Fig. 10 (a) shows the obtained temperature distribution of the heterogeneous object. As a comparison, the temperature of a homogeneous object (Aluminum, subject to the same boundary condition) is also provided in Fig. 10 (b). As can be seen the high temperature region of Fig. 10 (a) is smaller than that of Fig. 10 (b), this is because the heterogeneous object uses a less conductive primary material (Alumina) which helps to impede thermal conductions, as compared with the homogeneous object.

For brevity, Fig. 10 only shows the thermal conduction results of the heterogeneous object, however other coupled analysis (for instance, the thermal stress, strained energy density etc.) can be also conducted using similar approaches. A multi-physics based finite element analysis on 2D heterogeneous objects have been reported in our recent paper [27] and interested readers may find more technical details there.

The benefit of this integrated CAD modeling and FE analysis is that objects with complex heterogeneities can be easily analyzed with the finite element methods. Without proper CAD models, however, the complex material distributions can hardly be formulated or interrogated; further material property calculations and physical performance simulations are therefore very difficult to be obtained.

Using the integrated CAD modeling and FEA of heterogeneous object, the users can first design heterogeneous CAD models and then transfer the models to the CAE modules for property simulations. Modifications on the object geometries or material distributions can be carried out if the simulated properties do not satisfy the functional requirements. The modified CAD models can be further delivered to CAE modules to

re-validate its efficacies. The above process can be repeated until the users' functional requirements are fulfilled.

4 Rapid Prototyping of Heterogeneous Object

Once the designed heterogeneous object has been validated via numerical simulations, prototypes of the object can be then fabricated to test its physical behaviors under working conditions. In this section, Rapid Prototyping (RP) of heterogeneous objects is discussed and the manufacturing efficiency of objects with complex material heterogeneities is addressed.

4.1 Fundamental Algorithms

Typical processes involved in rapid prototyping of a heterogeneous object includes the following procedures [31]:

1. The object geometry is first sliced by parallel planes and a collection of boundary profiles are obtained, as shown in Fig. 11 (a) and (b);
2. For each slice, the silhouette boundary curves are covered into 2D regions, see Fig. 11 (c);

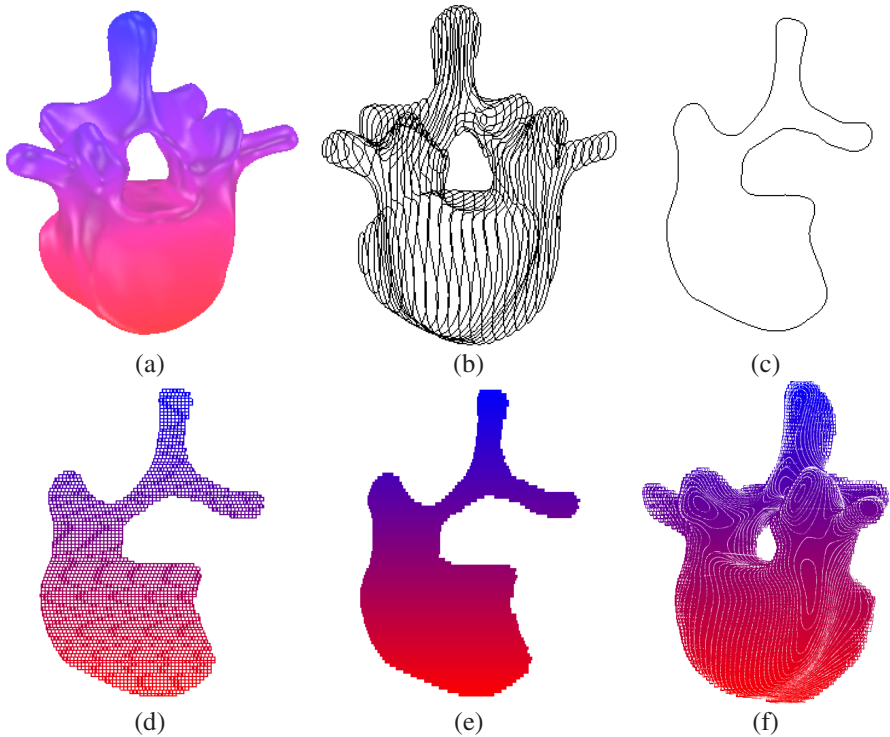


Fig. 11. Rapid prototyping of an example unidirectional FGM object

3. Each 2D region is intersected with scan lines and a collection of 1D solids is obtained;
4. Each scan line is further decomposed into an array of voxels;
5. For each voxel, its material composition is interrogated from the heterogeneous CAD model, as shown in Fig. 11 (d) and (e);
6. The obtained voxel data are used to drive the hardware setup (e.g. nozzles) to selectively deposit materials in a continuous point-wise, line-wise and slice-wise fashion until the object is completely fabricated, as shown in Fig. 11 (f).

4.2 RP Data Generation for Complex Heterogeneous Object

The aforementioned approach works well with objects with simple material distributions (for instance the unidirectional FGM object in Fig. 11), however if the objects under fabrication have complex material heterogeneities (e.g. the one shown in Fig. 5 (a)), simply applying the above discussed algorithms may introduce additional problems. As mentioned in Section 4.2, the part-assembly model is a widely used representation for objects with complex material heterogeneities; however, if the presented procedures in Section 4.1 are used in conjunction with the assembly model, significant robustness and efficiency problems may occur [31].

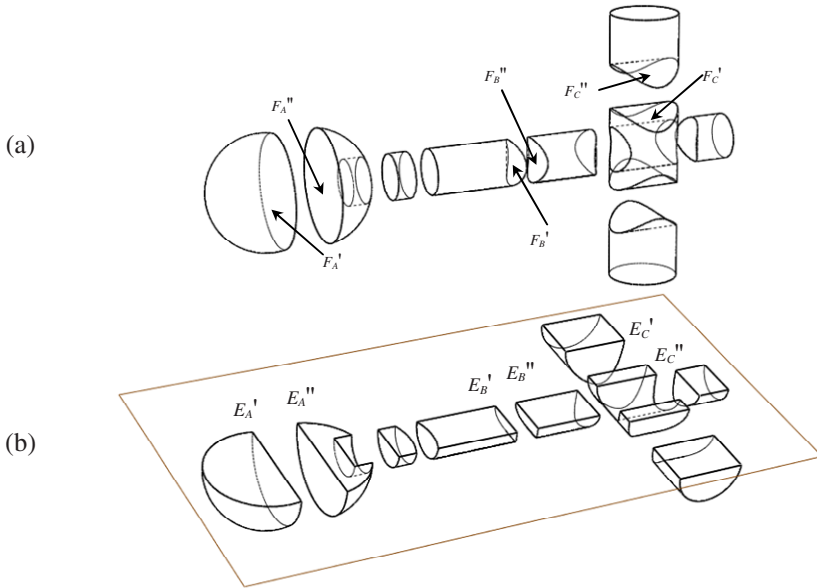


Fig. 12. Planar slicing of a part-assembly model in rapid prototyping of a complex heterogeneous object (a) Subdivided components of the object and redundant faces; (b) Redundant edges generated by repetitive planar slicing

Take the object in Fig. 5 (a) as an example. In the planar slicing stage, to get the boundary profile of the object, the slicing algorithm must be separately applied on all the nine components, as shown in Fig. 12. Note that many faces are repetitively kept in the assembly, for instance F_A' , F_A'' , F_B' , F_B'' and F_C' , F_C'' in Fig. 12 (a). All these redundant faces subsequently take part in the boundary slicing process, repetitive plane-face intersections are performed and redundant edges (e.g. E_A' , E_A'' , E_B' , E_B'' and E_C' , E_C'' in Fig. 12 (b)) are generated. Similarly in the line scanning process, identical line-edge intersections will be conducted on these redundant edges. Also note that these repetitive plane-face and line-edge intersection computations are ubiquitous in the entire RP data generation process and they are performed in *every* planar slicing and line scanning step. These repetitive and unnecessary boundary intersections therefore, greatly degraded the overall efficiencies.

A careful study into this problem shows that in most cases, these redundant entities (faces and edges) serve as the delimiting boundaries of some “sub-domains” [31], and each of such sub-domain has different material distributions [31]. They are introduced solely for the purpose of point membership classifications and material interrogations [22, 31, 32]. Conceptually, such material delimitation entities should not be included in the geometric operations (such as section slicing and line scanning); but rather, they should be utilized only in the material evaluation process (the Step 5 in Section 0).

To improve the computational efficiency in the RP data generation, these repetitively kept entities should be temporarily excluded from the boundary intersection computations; and only the boundary elements which bound the object geometries should actually participate in the plane-face, line-edge intersections.

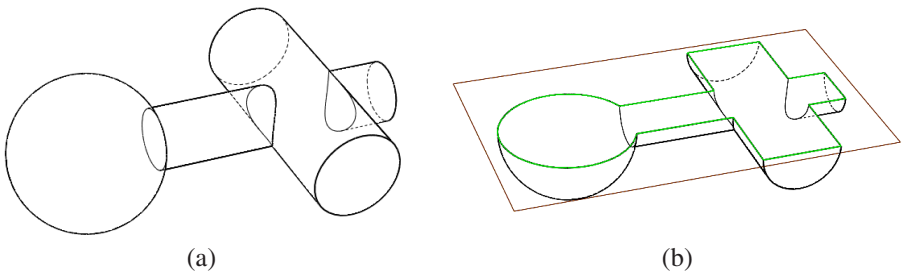


Fig. 13. Selective boundary slicing in rapid prototyping of a complex heterogeneous object

With the traditional part-assembly model, the redundant entity removal is almost unattainable since all the boundary elements are equally treated. By using the proposed HC-Rep, however, the unnecessary and repetitive boundary-interaction calculations can be avoided and this is accomplished through a selective boundary slicing algorithm [31]. In this algorithm, all the boundary faces are first retrieved from the non-manifold cellular models and kept in a face list; the internal material delimitation boundaries, which share themselves with other boundary elements (i.e. *double-sided*, see also Section 0) are then removed from the list. The remaining faces are subsequently sewn

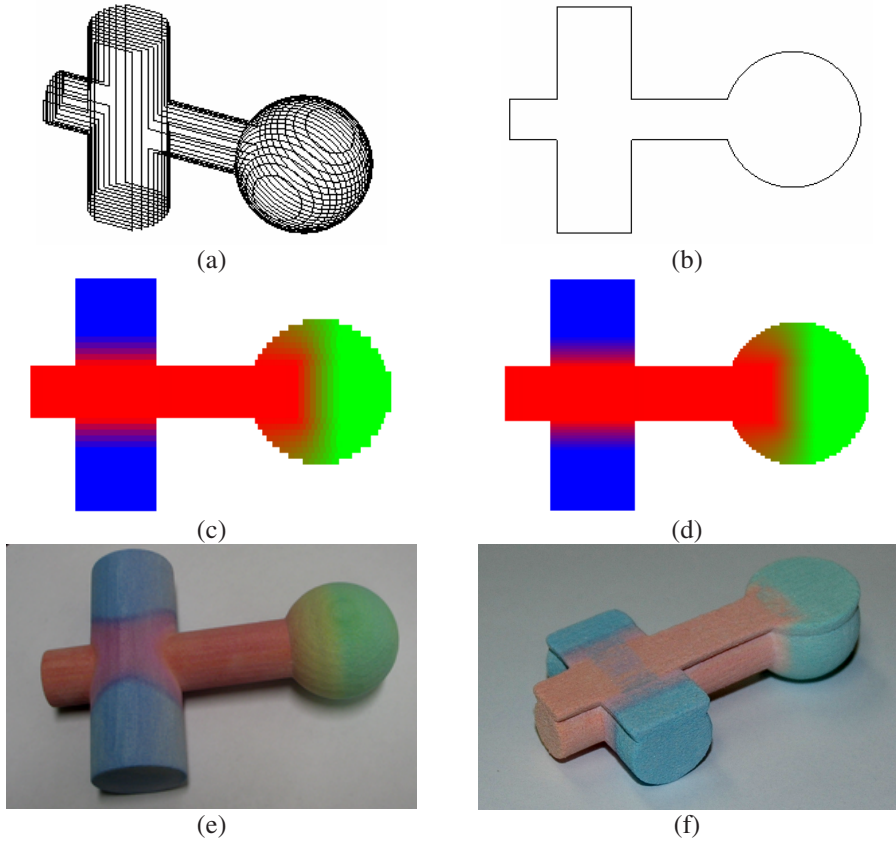


Fig. 14. Planar slicing of the manifold geometry and the layered representation of the complex heterogeneous object. (a) and (b): results of applying the selected planar slicing on manifold geometry; (c) and (d): layered representations of the generated data to be used in the RP process. (e) and (f): Prototypes fabricated with 3D Printer [33].

together to form a manifold solid, as shown in Fig. 13 (a). Note that it is the boundaries of the manifold solid that participate in the actual planar slicing and region covering, as shown in Fig. 13 (b).

Fig. 14 shows the results of applying the selective planar slicing on the *manifold* geometries. The sliced 2D regions are then scanned line by line, and further decomposed into an array of voxels, as described earlier. Based on the presented eHFT structure, the material composition of each voxel is then interrogated from the HC-Rep model.

The final layered representations of the generated data are shown in Fig. 14 (c) and (d), and the effects of different fabrication resolutions are demonstrated. Fig. 14 (e) and (f) show two prototypes fabricated using the Z Corporation's 3D printer [33]; the material distributions of both the outer boundary and internal parts are illustrated.

5 Implementations

The proposed integrated CAX based design approach is partially implemented in our standalone heterogeneous object modeler — CAD4D [34], jointly with COMSOL Multiphysics [30].

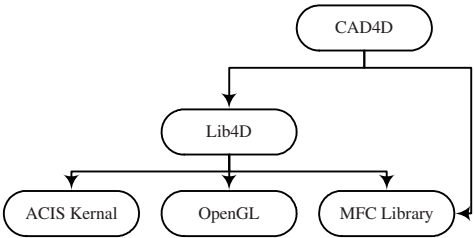


Fig. 15. Software implementation architecture of CAD4D

CAD4D is based upon a reusable object class library *Lib4D* [26, 34] and Microsoft Foundation Class (MFC) libraries. The commercial 3D kernel *ACIS* [28] is used for handling geometric modeling related issues, C++ Standard Template Library (STL) is used to implement container related data structures, and OpenGL is used as the rendering engine for object visualizations, as shown in Fig. 15.

In a typical heterogeneous object design process, the users invoke the CAD modeling tools through graphical user interfaces (e.g. clicking toolbar buttons or menu items). CAD4D performs modeling operations by manipulating relevant object data structures with proper algorithms. For instance, in object constructions, *Lib4D* object instances are created and appended to the object database (Fig. 16 (b1) and (c1)), visualizations are then updated in response of the user actions as shown in Fig. 16. Fig. 17 shows a snapshot of the proposed CAD4D modeler.

The integrated CAD modeling and finite element analysis are conducted using CAD4D and COMSOL Multiphysics, as detailed in Section 3.

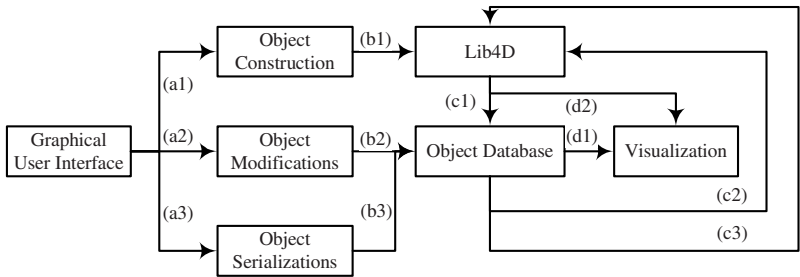


Fig. 16. Typical user interactions and modeling processes using CAD4D modeler

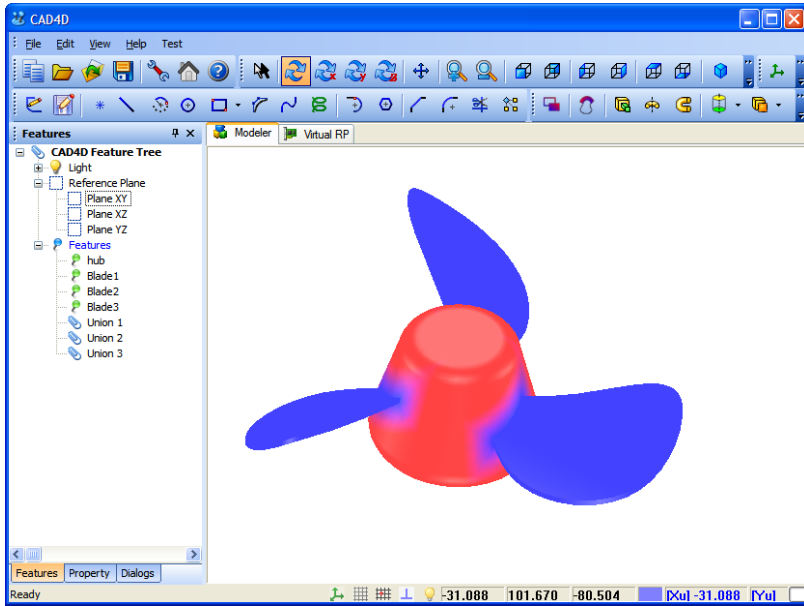


Fig. 17. A Snapshot of the CAD4D modeler

The integrated CAD and CAM of heterogeneous objects are carried out by data communications with CAD4D and Z Corporation's 3D printer, mostly via the VRML data format. A heterogeneous CAD model is designed by CAD4D and validated/verified by COMSOL Multiphysics; once the functional requirement of design is satisfied, the HC-Rep based CAD models are then converted to VRML format for physical fabrications.

6 Conclusions

This article presents an integrated CAX (CAD/CAE/CAM) perspective on heterogeneous object design. Different from many existing approaches which emphasize a particular aspect of the design problem, heterogeneous object design is envisaged as an integral process which combines the CAD modeling, property analysis and physical realization. The article is motivated to present such a perspective, demonstrate the typical design procedures, paradigms and benefits. The emphasis of this article is on the linkage and integration of the CAX tools, rather than each separate topic. The readers may, however, refer to our previous papers [18, 22], [27] and [31] for more technical details on each subject of interest. Flash animations are also available on the first author's website <http://web.hku.hk/~kouxy>. Interested readers may download them to get a rough idea of the relevant schemes before delving into the technical details.

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